

A Simple Dynamic Analysis of Government Debts and Deficits

Our goal is to create a simple theoretical model that can help us understand the factors or determinants of the dynamic path of the government debt/GDP ratio and the government deficit/GDP ratio.

All Analyses of Government Debt and Deficits begin with the
Government Budget Constraint

Total Government Expenditures = Total Government Revenues and Resources

which in highly simplified form becomes

$$rB + PG = \tau PY + \Delta B + \Delta M$$

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where r = Nominal Interest Rate

B = Nominal Value of Government Debt (with ΔB = change in the debt = the deficit)

P = General Price Level (with $\pi \equiv \Delta P / P$ = inflation)

G = Real Government Spending on Goods and Services

τ = Marginal Tax Rate on Income

Y = Real Income

M = Money Stock (with ΔM =change in money stock)

also let $b \equiv \frac{B}{PY}$, or the debt/GDP ratio.

In addition to the budget identity and our definitions of terms above, we have certain simplifying assumptions to keep the analysis manageable from the outset. These are as follows...

Assumptions

(1) $r = \rho_o + \pi$ where $\rho_o =$ constant at its "natural" level

(2) $\lambda_o \equiv \frac{\Delta Y}{Y}$ which is constant at the "natural" level

(3) $\theta_o \equiv \frac{\Delta M}{M}$ which is constant by assumption

(4) $MV_o = PY$ where V_o is constant by assumption

(5) $g_o \equiv \frac{G}{Y}$ is a policy parameter and is held constant

Analysis

Consider the government budget again -- $rB + PG = \tau_o PY + \Delta B + \Delta M$

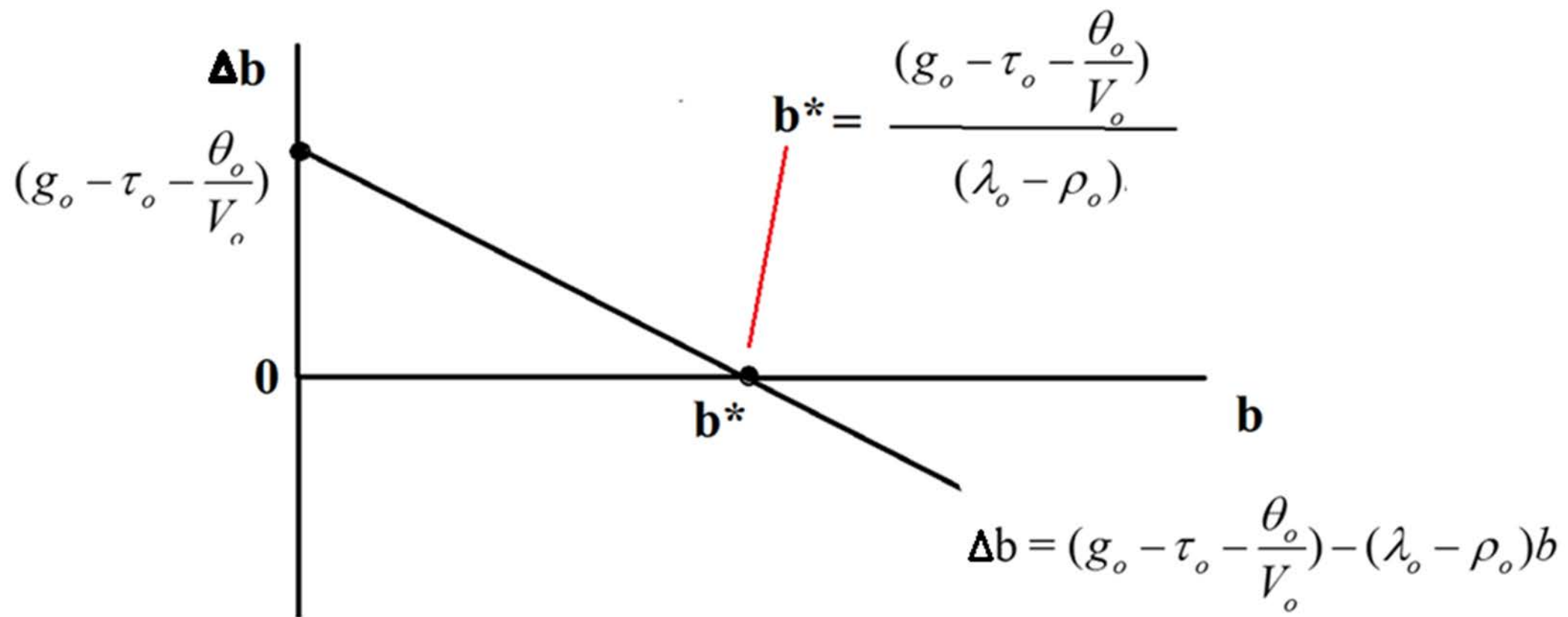
$$\Rightarrow r\left(\frac{B}{PY}\right) + (g_o - \tau_o) = \frac{\Delta B}{PY} + \frac{\Delta M}{M} \frac{M}{PY}$$

$$\Rightarrow rb + \left(g_o - \tau_o - \frac{\theta_o}{V_o}\right) = \frac{\Delta B}{PY}$$

but, it is easy to show that $\frac{\Delta B}{PY} = \Delta b + (\lambda_o + \pi)b$

$$\therefore (\rho_o + \pi)b + \left(g_o - \tau_o - \frac{\theta_o}{V_o}\right) = \Delta b + (\lambda_o + \pi)b \text{ and thus,}$$

$$\Delta b = \left(g_o - \tau_o - \frac{\theta_o}{V_o}\right) - (\lambda_o - \rho_o)b$$



The variable b^* is called the long run, or steady state, value of the debt to GDP ratio. We can write this explicitly as

$$\mathbf{b}^* = \frac{(g_o - \tau_o - \frac{\theta_o}{V_o})}{(\lambda_o - \rho_o)}$$

Each parameter on the right hand side has a clear economic meaning and has a definite effect on b^* .

Unfortunately, the world is not so simple. The parameters we have assumed are constant are actually quite variable. They are not constant.

It is also true that, in order to have a stable long run b^* , we must assume the economy grows faster than the real interest rate ($\lambda_o > \rho_o$) and some economists like T. Piketty have argued empirically that the reverse is true. He claims this is causing greater income inequality in the world.

Since our “fixed” parameters in the model are actually quite changeable, a controversy has arisen over the selection of the appropriate conduct of macroeconomic policy. In broad terms we can have more stimulus or greater austerity. More stimulus would increase g_0 and lower τ_0 . Greater austerity would do just the opposite. Austerity is sometimes called “fiscal consolidation”.

How can we talk about the stimulus vs. austerity debate within the context of the above model?

Return again to the steady state value of the debt to GDP ratio.

$$\mathbf{b}^* = \frac{(g_o - \tau_o - \frac{\theta_o}{V_o})}{(\lambda_o - \rho_o)}$$

Ostensibly, greater austerity (say a reduction in g_o) would lower b^* . But, suppose that reducing g_o reduces λ_o ? There is no reason then to believe that b^* would fall, and in fact, it may rise. In our simple model, we merely assumed λ_o was fixed, but it very well may depend on g_o .

Suppose that greater stimulus is tried. Opponents to this might argue that a rise in g_0 will fail to raise long run λ_0 and the debt/GDP will merely increase secularly. Eventually, the debt will become too large and the real interest rate ρ_0 will rise, causing the system to become unstable, with a severe inflation (i.e. rising θ_0) the only means of reducing the debt.

$$\mathbf{b}^* = \frac{(g_0 - \tau_0 - \frac{\theta_0}{V_0})}{(\lambda_0 - \rho_0)}$$

One part we have not discussed is why that V_0 is important to the determination of the long run debt/GDP ratio.

Write $MV_0 = PY$ in the alternative form of $M/P = Y/V_0$. Here we see that in equilibrium, Y/V_0 is just the real demand for money. The term $1/V_0$ is the proportion of our income we hold in the form of money. Now, consider the revenue the government gets from the inflation tax, which in the long run is

Nominal Tax from Money Creation = $\theta_0 M$ and dividing by PY gives

Ratio of Revenue from Money Creation to GDP

$$= \theta_0 M/PY = \theta_0/V_0 \text{ in equilibrium}$$

If money creation is an alternative to issuing interest bearing debt, then a greater demand for money (i.e. a smaller V_0) will allow the government to issue money faster and allow it to reduce its issuing of interest bearing debt so quickly. This will naturally reduce the debt to GDP ratio over the long run.

Finally, we might ask what happens to the deficit to GDP ratio in the long run. Again, we can use this model to answer this question.

In our model, the deficit to GDP ratio is written as

$$\frac{\Delta B}{PY} = \Delta b + (\lambda_o + \pi)b \quad \text{But, } \Delta b \text{ is zero in the long run, and } \pi = \theta_o - \lambda_o.$$

$$\text{Therefore, in the steady state } \left(\frac{\Delta B}{PY}\right)^* = \theta_o b^* = \frac{\theta_o (g_o - \tau_o - \frac{\theta_o}{V_o})}{\lambda_o - \rho_o}.$$

The long run deficit to GDP ratio will behave much like the debt/GDP ratio except for the case of a change in the growth rate of the money supply.