Chapter 2

Consumer Choice and Demand

The last chapter set up just one-half of the fundamental structure we need to determine consumer behavior. We must now add to this the consumer's budget constraint, which in turn forces the consumer to make decisions based on his or her opportunity costs. Taking the indifference curves together with the budget constraint, we will be able to determine the demand for each good in the consumer's utility function.

So, what is the budget constraint? This depends on how sophisticated we want to make the analysis. Ignoring work effort, wealth and saving, along with future consumption and future utility, we can write the budget constraint for an individual and two goods as

$$I = P_1X_1 + P_2X_2$$

where $I =$ income, $P_i =$ price of the ith good, $X_i =$ quantity of ith good bought and consumed. It is assumed that income and prices are exogenous, meaning that they are not controlled by the individual. This individual chooses X_1 and X_2 so as to maximize the total utility generated by the $X's$. This is summarized by saying that the individual solves the decision problem

$$\underset{X_1 X_2}{Maximize}\ U(X_1, X_2) \quad \text{subject to } I = P_1X_1 + P_2X_2$$

It is important to realize that we are not saying that people actually go through this mathematical optimization in solving their decision problems. We are merely developing a model that hopefully mimics human decision-making. The model is sometimes called the "rational model" of decision-making.

Holding the income level constant, along with the prices, we can consider the budget constraint and the indifference mapping together. This is done in the graph below.

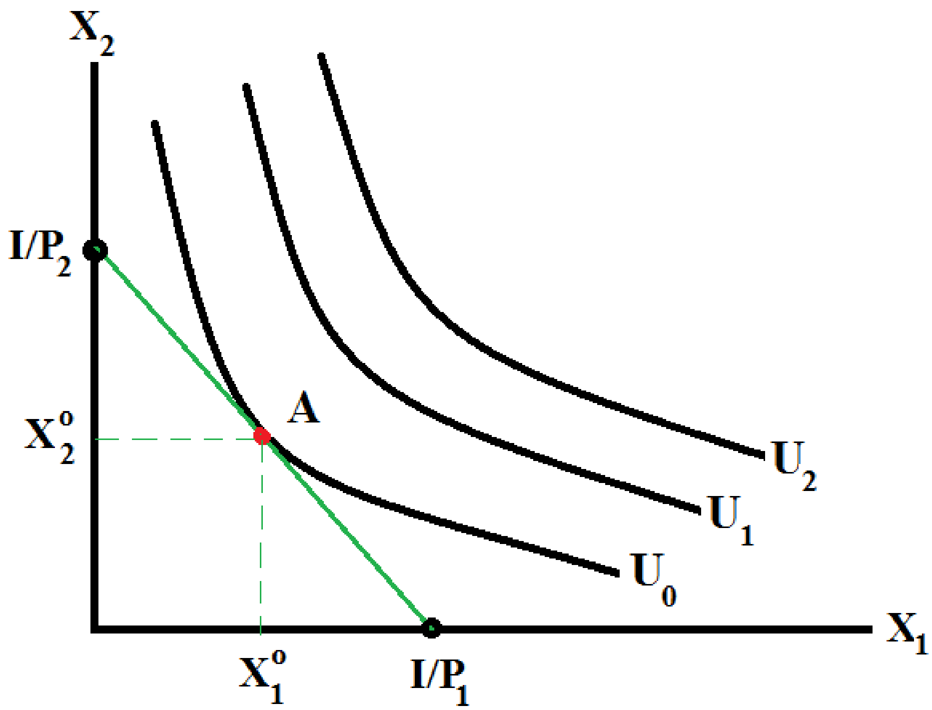
Note that the points in the graph corresponding to I/P_1 and I/P_2 are the maximum amounts of X_1 and X_2, respectively, that can be bought, given the income and prices. The red point A is the consumer equilibrium point corresponding to highest utility. It is impossible for the consumer to choose <u>any</u> other point, satisfy the budget constraint, and still have higher utility. This is because every other point on the green budget line lies on a lower indifference curve.

What is the slope of the budget line? Holding prices and income constant we can change the quantities X_1 and X_2. This can be written as

$$0 = P_1 \Delta X_1 + P_2 \Delta X_2$$

Rearranging terms we can write the slope of the budget line as

$$\text{Slope of Budget Line} = \left(\frac{\Delta X_2}{\Delta X_1}\right)_{Budget} = -\frac{P_1}{P_2}$$



What about the slope of the indifference curve? Here we take the utility function and hold U constant while letting the quantities X_1 and X_2 change. Doing this, we can write

$$0 = \left(\frac{\Delta U}{\Delta X_1}\right)\Delta X_1 + \left(\frac{\Delta U}{\Delta X_2}\right)\Delta X_2 = MU_1 \Delta X_1 + MU_2 \Delta X_2$$

from which it follows that

$$\text{Slope of Indifference Curve} = \left(\frac{\Delta X_2}{\Delta X_1}\right)_{Indiff} = -\frac{MU_1}{MU_2}$$

Putting the two slopes equal at equilibrium gives us the condition

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

Sometimes this is written in a slightly different form as

$$\frac{MU_1}{P_1} = \frac{MU_2}{P_2}$$

How can we interpret this last equation? The additional utility acquired per dollar of income spent should be the same across all goods. If this condition is not met, then it will be possible to rearrange expenditure and raise utility.

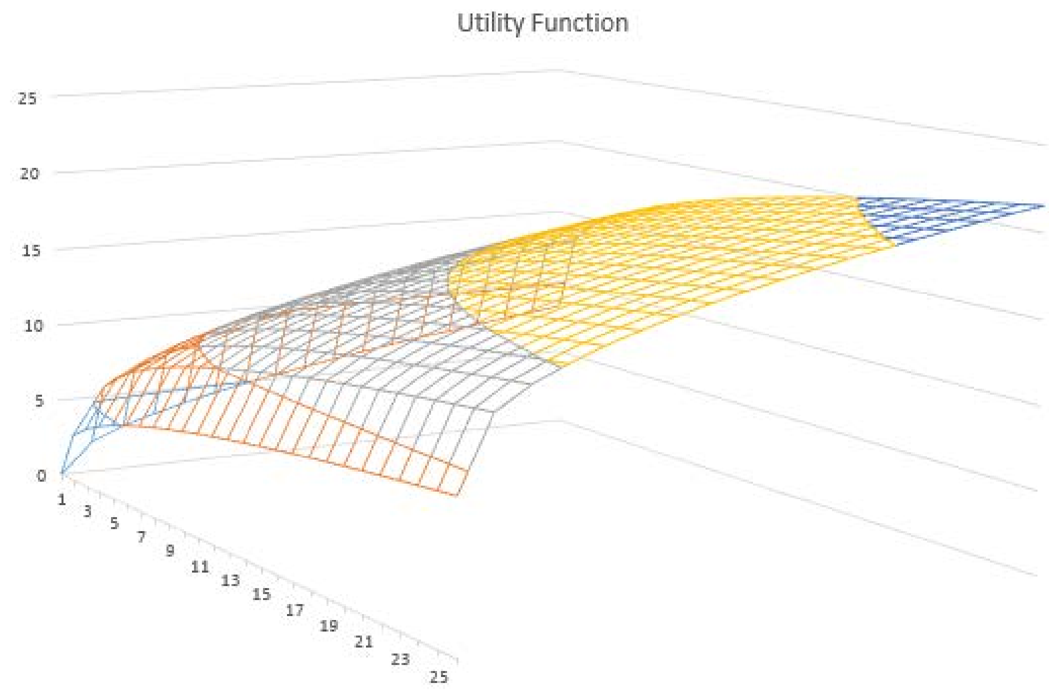
Below is a screenshot of an Excel sheet that calculates a utility surface given by

$$U(X_1, X_2) = 2(X_1)^{1/2} + 3(X_1)^{1/3} + 4(X_1 X_2)^{1/4}$$

Each entry is the value of U for the X_1 and X_2 column and row. I have then approximately traced out three indifference curves. Note how that along the green indifference curve the value stays close to 14.5. An actual and exact indifference curve would be exactly equal to 14.5 at any point on the indifference curve.

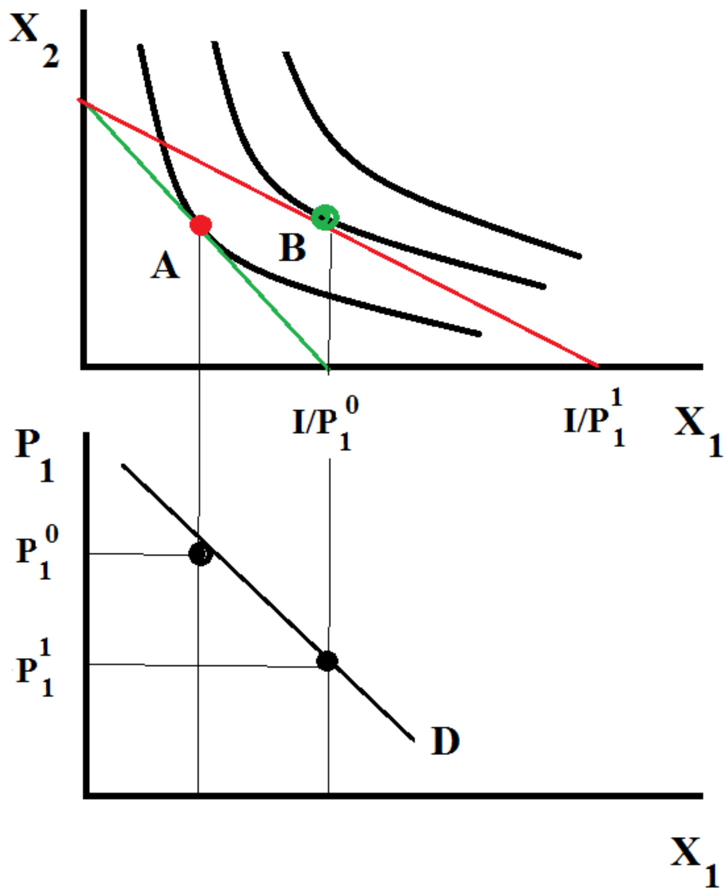
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1																				
2											X1									
3			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4		0	0	2	2.82843	3.4641	4	4.47214	4.89898	5.2915	5.65685	6	6.32456	6.63325	6.9282	7.2111	7.48331	7.74597	8	8.24621
5		1	3	6	7.01763	7.78018	8.41421	8.96748	9.46406	9.91808	10.3386	10.7321	11.1028	11.4544	11.7894	12.1099	12.4177	12.714	13	13.2768
6		2	3.77976	6.96897	8.0224	8.80895	9.46156	10.0302	10.54	11.0056	11.4366	11.8395	12.2191	12.5787	12.9213	13.249	13.5634	13.8661	14.1582	14.4407
7		3	4.32675	7.64282	8.72026	9.5229	10.188	10.7669	11.2855	11.7589	12.197	12.6063	12.9917	13.3568	13.7044	14.0369	14.3558	14.6627	14.9589	15.2453
8		4	4.7622	8.17642	9.27242	10.0875	10.7622	11.3491	11.8745	12.354	12.7975	13.2117	13.6016	13.971	14.3226	14.6587	14.9811	15.2913	15.5906	15.88
9	X2	5	5.12993	8.62528	9.73663	10.562	11.2447	11.8381	12.3693	12.8537	13.3016	13.7199	14.1136	14.4864	14.8413	15.1804	15.5058	15.8187	16.1206	16.4125
10		6	5.45136	9.01645	10.141	10.9752	11.6647	12.2638	12.7998	13.2886	13.7404	14.1622	14.5591	14.9349	15.2925	15.6343	15.9621	16.2774	16.5815	16.8755
11		7	5.73879	9.36537	10.5016	11.3436	12.0391	12.6432	13.1835	13.676	14.1312	14.5561	14.9559	15.3343	15.6944	16.0385	16.3685	16.6858	16.9919	17.2878
12		8	6	9.68179	10.8284	11.6775	12.3784	12.987	13.5311	14.0271	14.4853	14.913	15.3153	15.6961	16.0584	16.4045	16.7365	17.0557	17.3636	17.6612
13		9	6.24025	9.9723	11.1284	11.9839	12.6897	13.3024	13.85	14.3491	14.8101	15.2403	15.6449	16.0278	16.3922	16.7402	17.0739	17.3949	17.7044	18.0035
14		10	6.4633	10.2416	11.4065	12.2678	12.9782	13.5946	14.1454	14.6473	15.1109	15.5434	15.9501	16.3351	16.7013	17.0511	17.3864	17.7089	18.0199	18.3204
15		11	6.67194	10.4931	11.6661	12.5328	13.2474	13.8673	14.4212	14.9257	15.3916	15.8263	16.235	16.6218	16.9897	17.3411	17.678	18.0019	18.3143	18.6161
16		12	6.86829	10.7295	11.9101	12.7819	13.5004	14.1236	14.6802	15.1872	15.6553	16.092	16.5026	16.8911	17.2606	17.6135	17.9518	18.2771	18.5907	18.8938
17		13	7.054	10.9528	12.1405	13.0171	13.7394	14.3656	14.9248	15.4341	15.9043	16.3429	16.7552	17.1453	17.5163	17.8707	18.2103	18.5368	18.8517	19.1559
18		14	7.23043	11.1648	12.3592	13.2403	13.966	14.5951	15.1568	15.6683	16.1404	16.5808	16.9948	17.3864	17.7588	18.1145	18.4554	18.7831	19.0991	19.4044
19		15	7.39864	11.3666	12.5674	13.4528	14.1818	14.8136	15.3777	15.8912	16.3652	16.8073	17.2228	17.6159	17.9897	18.3466	18.6887	19.0176	19.3346	19.6409
20		16	7.55953	11.5595	12.7664	13.6558	14.388	15.0224	15.5887	16.1042	16.58	17.0236	17.4406	17.8351	18.2101	18.5683	18.9115	19.2415	19.5595	19.8668
21		17	7.71384	11.7444	12.957	13.8503	14.5855	15.2224	15.7908	16.3082	16.7857	17.2308	17.6493	18.045	18.4213	18.7806	19.1249	19.4559	19.7749	20.0832
22		18	7.86222	11.922	13.1401	14.0371	14.7752	15.4144	15.9849	16.5041	16.9832	17.4298	17.8496	18.2466	18.6241	18.9845	19.3298	19.6618	19.9818	20.2909
23		19	8.0052	12.093	13.3165	14.217	14.9578	15.5993	16.1718	16.6927	17.1733	17.6214	18.0424	18.4407	18.8192	19.1807	19.527	19.8599	20.1808	20.4908
24		20	8.14325	12.258	13.4865	14.3905	15.134	15.7777	16.352	16.8745	17.3567	17.8061	18.2284	18.6278	19.0074	19.3699	19.7172	20.051	20.3727	20.6835
25		21	8.27677	12.4175	13.6509	14.5582	15.3042	15.95	16.5261	17.0503	17.5338	17.9846	18.4081	18.8086	19.1893	19.5527	19.9009	20.2356	20.5582	20.8698
26		22	8.40612	12.5719	13.8101	14.7205	15.4689	16.1168	16.6947	17.2204	17.7053	18.1573	18.582	18.9835	19.3652	19.7296	20.0787	20.4142	20.7376	21.05
27		23	8.5316	12.7215	13.9643	14.8778	15.6286	16.2785	16.858	17.3852	17.8715	18.3247	18.7505	19.1531	19.5357	19.901	20.251	20.5873	20.9115	21.2246
28		24	8.6535	12.8669	14.1141	15.0305	15.7837	16.4354	17.0166	17.5452	18.0328	18.4872	18.914	19.3176	19.7012	20.0674	20.4182	20.7553	21.0802	21.394
29		25	8.77205	13.0081	14.2596	15.179	15.9343	16.5879	17.1707	17.7007	18.1895	18.645	19.073	19.4775	19.862	20.2291	20.5807	20.9186	21.2442	21.5587
30																				

What does the utility surface look like in 3-d (X_1, X_2, U) space? Here is the 3-d surface rendering done by Excel below. Note how it looks like a smooth hill sloping ever upward at a declining rate. Although there are only a few colors, the height of the surface is continually increasing as the quantities X_1 and X_2 increase.



How can the demand for X_1 be determined from this graphical analysis above using the indifference curves and the budget line? We first assume income, I, is held constant. The price of P_1 is then lowered one time and a new equilibrium is attained. In our graph below this is represented by a movement from the red point at point A to the green point at point B. Obviously, from a comparison of the two points we can say that the lowering of the price P_1 results in a greater quantity demand of X_1 and a smaller quantity demanded of X_2. Note that the quantity demanded of X_2 occurs even though P_2 remains constant throughout. This is because the demands for X_1 and X_2 depend on the relative price (P_1/P_2). Only one of the prices needs to change for both demands to be affected. There are other outcomes that are possible, as will be shown in class. Later in the class, we will show how that a price change can be decomposed into an income effect and a substitution effect. You might want to try to draw the indifference curves in such a way that the demand for X_2 is unaffected by a fall in the price of X_1. Be sure that your indifference curves are convex to the origin and that they do not touch or slope upward.

Can indifference curves change slope or shift positions? We have said that if we allow preferences to change, it is impossible to say anything clear about changes in the observable data on prices and quantities. Changing the indifference curves mean that our careful analysis of price changes and income changes on demand is lost. Therefore, economists are loath to talk about such things. However, it cannot be denied that preferences change as one gets older. When one is young there is certainly no need for a cane, but this is not true when one becomes elderly. More broadly, the importance of health care in the utility function must be greater for the old than for the young. The introduction of new products can also affect the utility function. It is useless to ask what the demand for smart phones were in 1950. But, the question is very important today. It is similarly true that as one's body begins to deteriorate, many of the good and services that were important when young become less important. Thus, preferences are a clear matter of importance to economists. Because of this, economists have claimed that preference change much more slowly than prices and income. This difference in the speed of change makes a huge difference, since it lets us separate the fast moving changes (prices and incomes) from the slow moving changes (demographics, and preference changes).



How can we depict the effect of a change in income on the demand for X_1, a normal good (i.e. a good whose demand increases when consumer income increases)? The figure below shows this for a normal good. A rise in income from I_o to I_1 shifts the budget line out parallel. The slope doesn't change because prices do not change. Thus, the equilibrium shifts from point A to point B. This means that at some unchanged price, the quantity of X_1 increases. This is shown in the bottom frame where the demand moves from the black point to the red point. Since we fixed the price of P_1 arbitrarily, we know that the whole demand curve shifts to the right. You should consider the case where income rises and there is no shift in demand. You must be clever about drawing the indifference curves. Finally, it will be possible that an increase actually shifts demand to the left. Try to show this for a certain collection of indifference curve. Remember, you must draw downward sloping indifference curves that are convex to the origin. They cannot slope upward. They cannot touch each other. If you can draw a set of these, then a utility function will be defined by them.