Production, Revenue, and Cost

All businesses are formed to produce a set of goods and/or services for sale at a profit. Business can be classified into proprietorships, partnerships, and corporations. The types of business organization chosen will be determined by how complicated the production and sale processes are for the good or service. In this chapter, we want to look at the fundamental aspects of producing and selling a good and how profits are determined. We will also want to explain why profits are important by emphasizing the function profits play in the economy.

Production is a technical relation between inputs and outputs. Over time this technical relation can be expected to change. However, the manner of change in technology is quite evolutionary and cannot be predicted well. A general principle would be that technological change is a response to the relative cost of the inputs used to produce the good or service. The more expensive an input is, the more likely a technological breakthrough will be achieved to reduce this input cost. For the most part, we will assume that technology is unchanged and inputs produce outputs according to a fixed rule. It will be convenient to think of this fixed rule relating inputs to outputs as a function, in which case we focus on multiple inputs and one output. This function is referred to as the production function.

In studying the broad aspects of production, cost, revenue, and profit, one typically looks at two important inputs (labor and capital) and one sees how these are combined by the firm to produce an output for sale. Here we mean machines, tools, equipment, plant, etc. when we used the term capital. Labor is measured in terms of labor hours of employment, while capital is also measured in terms of the time used or employed. It will be simpler if we assume capital is fully utilized at some fixed level. Thus, in the short run, only labor can be varied and thus output will respond entirely to this change in the employment of labor. In fact, this is not how things work in the real world, even in the short run. A firm will always be able to reduce the usage of its capital (i.e. machines and plant) and hence save on its energy costs (another variable factor in the short run we are failing to consider). The firm will not be able to escape the full cost of its capital however. While the cost of labor is a variable wage bill depending on labor hours and the wage rate, capital costs are fixed in the short run regardless of how much of the firm's capital stock is used.

Let's take a simple example of a production function in the short run to see how things work.

Suppose we let L = labor employed and K = capital employed (fixed). Next, call Y the level of output created using L and K. Suppose further that the relationship between the inputs (L&K) and the output Y is given by

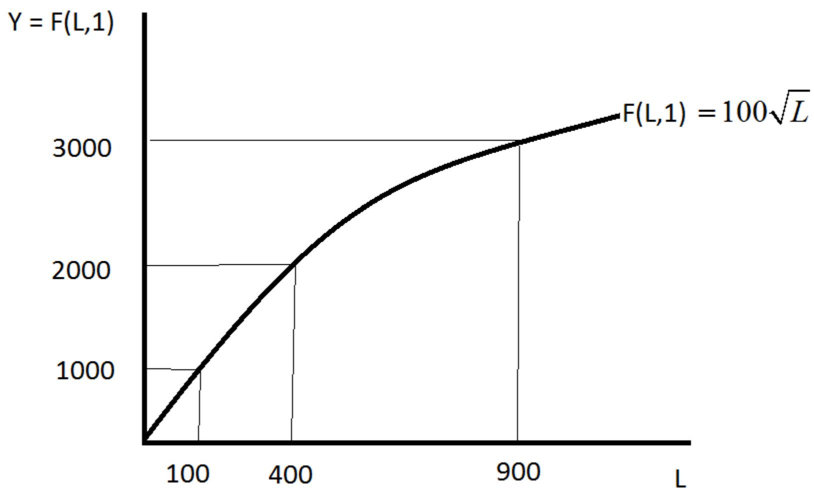
$$Y = F(L,K) = 100\sqrt{LK}$$

Furthermore, we can just let the fixed level of capital be equal to 1 to simplify things. Therefore, we have

$$Y = F(L,1) = 100\sqrt{L}$$

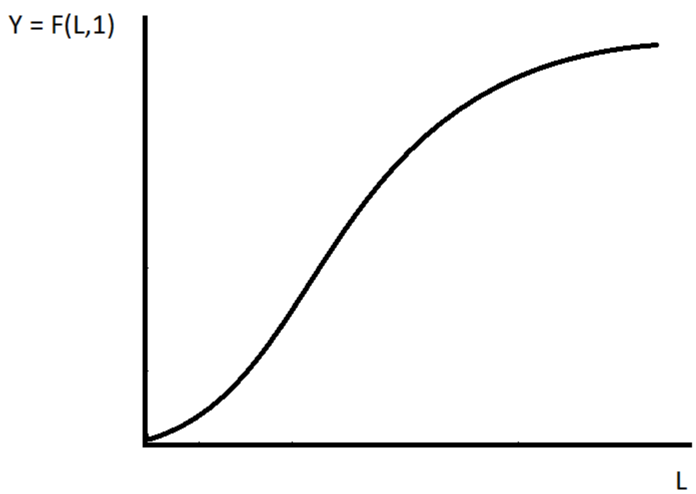
We can draw a graph showing this production technology (i.e. production function).

Figure 1 -- A Simple Production Function



Note how that in Figure 1 above the level of output rises as L increases, but the rate it rises decreases. The slope of the production function is decreasing as L increases. This is called diminishing marginal productivity of labor. This occurs because we have a fixed level of capital (machines) and therefore as we add more and more labor to a fixed level of machinery we get higher output at a falling rate. Just think of a room as the fixed level of capital and then keep adding more and more workers to the room. Eventually things get pretty crowded and it will be natural that the congestion causes less output to be produced. It may be that these congestion effects do not immediately kick-in. In that case, we can get a graph for our production function that looks like Figure 2 below. Output first rises at an increasing rate and then changes to a decreasing rate. This gives it the distinctive S-shape.

Figure 2 -- A Production Function with Delayed Diminishing Marginal Productivity



Both curves, those in Figures 1 and 2, represent the combining of inputs to produce output. In fact, the curve marks the largest output that can be produced using those inputs. The only way to raise the curve upwards is to get more and better capital or find a better way of combining the inputs to produce output. In the short run, we feel this will not happen, or at least will not be significant.

The output, Y, being produced may be a service and not a good. For example, in the short run, a taxi ride (measured in distance or time) could be the output. The car used would represent the capital, K, of the firm and the driver's labor time would be labor, L. Technology in this case would be the driving rules and skills that the driver has that are used to start the car, put it in gear, and navigate the best possible route. The roads and bridges are social capital which society provides this firm (not entirely free) and which augment and improve the private capital including the car, gas, motor oil, home base, and communications gear. Production by firms, even simple enterprises, can be quite detailed and complex.

What creates order out of this mess of details? The answer is that the consumer is guided by getting the best value for his or her money spent. The firm is similarly guided by producing at optimum levels producing the highest profits to the owners of the firm. But how do we define profit?

To answer this, we note that profit is defined as total revenue minus total cost. This can be written as

$$\pi = TR - TC$$

Consider total revenue first. The firm will sell its output Y at price P. Thus, its total revenue can be written as TR = PY. The firm will not receive any other revenue. That's it.

Now, we must turn to cost. Total costs are more complicated. In the short run, some costs are fixed and must be paid regardless of how much output is produced. Produce 1 unit of output or a million units of output, fixed costs do not change. Let's call these costs TFC, for total fixed costs. TFC include things like the rent paid on the buildings used, any interest on debts that have been incurred, the salaries that must be paid to administration, pensions to retirees, depreciation charges on the capital of the firm and well, just about any expense that is unavoidable in the short run. Next, add all costs that depend on the level of production. We call these total variable costs, or TVC. Any expense that depends on how much Y is produced is part of TVC. Think of the wages paid to labor and the cost of raw materials including energy costs. Remember, in the real world, running a fixed number of machines a variable amount of time involves a variable cost due to the energy expended. We will not consider these complications here, but they should be kept in the back of our minds at all time.

In simple treatments of cost, we have TFC (interest on the debt of the firm and possibly depreciation charges) and TVC (mainly the wage bill determined by multiplying the wage rate times the number of hours worked). Thus, short run profit can be written as

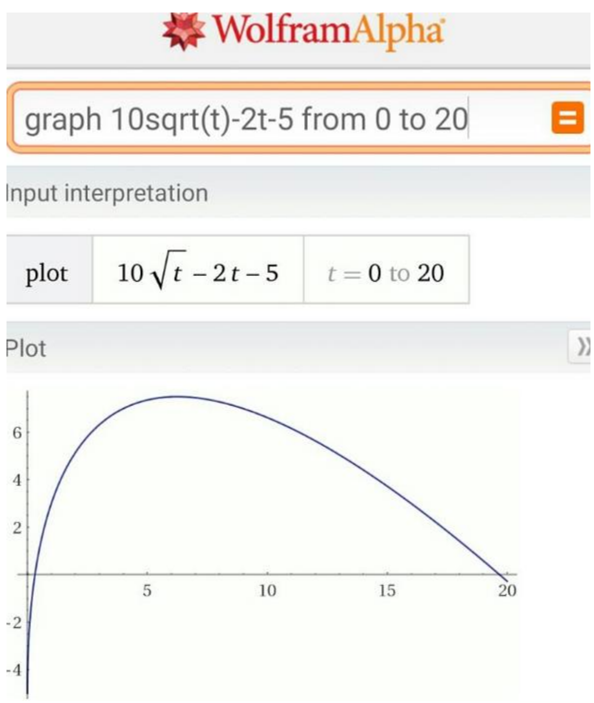
$$\begin{aligned}\pi &= TR - TC \\ &= PY - (TVC + TFC) \\ &= PY - wL - rK \\ &= PF(L,K) - wL - rK\end{aligned}$$

There are numerous problems with this simple formulation (e.g. we have equated nominal and real capital), but let's set these aside to better understand the basic structure of profits. Substituting in our simple example of a production function we have

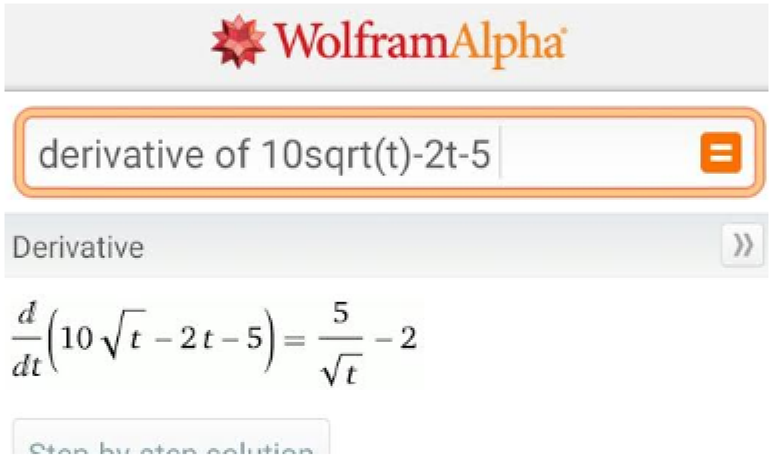
$$\begin{aligned}\pi &= PF(L,K) - wL - rK \\ &= P\sqrt{L} - wL - r\end{aligned}$$

We have drawn this profit function in Figure 3 below (using Wolfram Alpha to plot the profit function) where P = 10, w = 2, and r = 5.

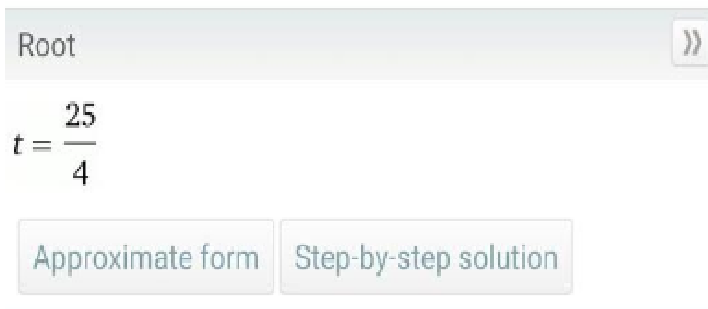
Figure 3 – The Profit Function for $F(L,1) = \sqrt{L}$



Note that maximum profit, the highest point on the graph in Figure 3, occurs at a level of employment of labor, L = 25/4 = 6.25. You can see this again by using your phone and Wolfram Alpha. Here is the way you do it.



You will also see this after it computes the roots



This means that profit maximizing production occurs where $Y=\sqrt{6.25}=2.5$. Total revenue will be TR = PY = (10)(2.5) = 25 and total cost will be TC = TVC + TFC = (2)(2.5) + 5 = 10. Profit = π = TR – TC = 25 – 10 = 15.

We can conceivably do the process above for any type of production process. We can also make it more complicated and complete by adding other factors in addition to labor and capital. The basic logic remains. We are determining the level of output that produces the highest profit and we are relating this outcome to the price of the product and the level of the wage rate. In fact, the production function we have chosen allows us to write the short run supply curve for any level of price and wages. How can we do that?

Maximization of profit can be written as

$$\pi(L)=P\sqrt{L}-wL-TFC$$

At the point of maximum profit, it should be true that raising or lowering the level of L should result in no change in the profit (since we are at the top of the profit curve in Figure 3. Now we know that $Y=\sqrt{L}$ which means that $Y^2=L$. This implies that $2Y\Delta Y=\Delta L$ and therefore when $Y=\sqrt{L}$ it is true that $\frac{\Delta Y}{\Delta L}=\frac{1}{2\sqrt{L}}$. It is easy now to see how that profits change with changes in L at the maximum point.

$$\frac{\Delta Y}{\Delta L}=P\frac{1}{2\sqrt{L}}-w=0$$

So, the optimum level of labor is easily seen to be $L^{*}=\frac{P^{2}}{4w^{2}}$ and the supply of Y is $Y=\sqrt{L^{*}}=\frac{P}{2w}$. Of course, given a fixed wage and variable price P, the supply curve is easy to draw. Note how that a rise in the wage rate reduces supply, just as we would have supposed.

Lastly we might want to ask why there are profits? Why not simply distribute profits to labor? The usual explanation for profits is that before one enters the market there must be an incentive to invest and risk one's wealth in a business. Labor does not risk its wealth. Of course, a laborer can at anytime lose his or her job, but they nevertheless retain the wealth they have. It is only when they take their own capital and invest it in the business that they are subject to the same risk as the owner. The riskier the business the more profit will be required for the owner to risk part of his wealth. Remember that in modern economies, investors include huge pension funds and insurance companies managing the small funds of literally millions of people. It is the small investors that require a compensation of the risk they take on. Not all wealth is so concentrated as is sometimes asserted. Billions of dollars in insurance and pension funds may be concentrated in a relatively small number of owners of an insurance company or investment fund, but these funds are not real net wealth. They only represent assets covering enormous liabilities and potential payouts. To truly understand the distribution of wealth and thus risk, one must look at the distribution of liabilities as well. Profits are the compensation for the risking of one's wealth and everyone with wealth understands this.

(P1) What is the definition of profit?

(P2) What is the definition of total revenue?

(P3) Why are there fixed and variable costs in total cost?

(P4) Give an example of a variable cost. Now do the same for fixed cost.

(P5) What is a production function and what are the usual factors we find in production?

(P6) Why do we have profits in business?

(P7) Draw the supply curve $Y=\frac{P}{2w}$ where w = 3.